A New Solvable Procedure for Parametric Linear Complementarity Problem with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers Approach

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Abstract: The Parametric linear complementarity problem (LCP) and its importance are well known. In this paper, a new algorithm for solving parametric linear complementarity problems (PLCP) with Symmetric Trapezoidal Intuitionistic fuzzy number coefficients is proposed. A numerical example is given to illustrate the efficiency of the proposed method. The proposed method is easy to understand and apply.

Keywords: Fuzzy Linear Complementarity problem, Parametric linear complementarity problems, Principal pivoting method, Symmetric Trapezoidal Intuitionistic fuzzy numbers

I. INTRODUCTION

The linear complementarity (LC) problem is one of the most widely studied problems of mathematical programming since it arise in a variety of applications [3, 4, 6] in engineering, economics and applied sciences. Several methods have been proposed for solving LC problems. Iterative methods for the solution of LC problem were considered in [2]. Yassine discussed a comparative study between Lemke's method and the Interior point method for the monotone linear complementarity problem

A Fuzzyquadratic programming (QP) problem is a special case of non- linear programming problem. The two important methods for solving QP problem are Wolfe's method and Beale's method [1]. Wolfe used Kuhn Tucker conditions to transform the QP problem into linear inequalities and complementarity slackness. Also, FQPP can be transformed into FLCP using the KKT conditions and then, it can be solved by Lemke's algorithm.

In this paper, we propose a new method namely, algebraic elimination method for finding a complementarity feasible solution to the FLCP. In the algebraic elimination method, we first reduce the FLCP into an inequality system by using the relation $W \geq 0$ and then, the resulting reduced inequality system is solved by matrix / algebraic method. The proposed method for solving FLCP is very simple and easy to understand and also, to apply. Further, we extend the algebraic elimination method for solving Fuzzy quadratic programming problems with linear constraints after converting to FLCP. Numerical examples are given for better understanding of the solution procedures of the proposed method.

II. PRELIMINARIES

2.1 Fuzzy set

A Fuzzy set Ã is defined by Ã = {x,μ_A(x)}; x€A, μ_A(x)€[0,1]. In the pair (x, μ_A(x)), the first element x belong to the classical set A, the second element μ_A(x), belong to the interval [0,1] called membership function.

2.2 Symmetric Trapezoidal intuitionistic fuzzy number (STIFN)

A Symmetric Trapezoidal intuitionistic fuzzy number (STIFN) is an intuitionistic fuzzy set in R with the following function and non-membership function .

$$\mu_{\tilde{A}^I} = \begin{cases} 0 & , x < a_1 - h \\ \frac{x-(a_1-h)}{h} & , a_1 - h \le x \le a_1 \\ 1 & , a_1 \le x \le a_2 \\ \frac{(a_2+h)-x}{h} & , a_2 \le x \le a_2 + h \\ 0 & , a_2 + h \le x \end{cases} \qquad \mu_{\tilde{A}^I} = \begin{cases} 1 & , x < a_1 - h' \\ \frac{x-(a_1-h')}{h'} & , a_1 - h' \le x \le a_1 \\ 0 & , a_1 \le x \le a_2 \\ \frac{(a_2+h')-x}{h'} & , a_2 \le x \le a_2 + h' \\ 1 & , a_2 + h' \le x \end{cases}$$

Where $a_1 \le a_2$ and $h,.h' \ge 0$. This STIFN is denoted by $\tilde{A}^I = \{(a_1, a_2, h, h); (a_1, a_2, h', h')\}$. for our Convenience. STIFN is denoted by $\tilde{A}^I = (a_1, a_{2,} h, h')$ throughout this paper.

Definition: 2.3. Modified arithmetic Operations on Symmetric trapezoidal intuitionistic fuzzy numbers

(STIFNS) Let $\tilde{A}^I = (a_1, a_2, h, h')$ *and* $\tilde{B}^I = (b_1, b_2, k, k')$ be two Symmetric trapezoidal intuitionistic fuzzy numbers. Then **(i) Addition:** $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, h + k, h' + k')$

(ii) Subtraction: $\tilde{A}^I - \tilde{B}^I = (a_1 - b_2, a_2 - b_1, h + k, h' + k')$

(iii) Multiplication: $\tilde{A}^I \times \tilde{B}^I = (\frac{a_1+a_2}{2} \times \frac{b_1+b_2}{2} - W), (\frac{a_1+a_2}{2} \times \frac{b_1+b_2}{2} - W.|w - w'|.|w - w_1'|)$

Where $w = \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) - \min(m).\max(m) - \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right)$

$$w' = \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) - \min(n).\max(n) - \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right)$$

$$w_1' = \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right) - \min(n').\max(n') - \left(\frac{a_1+a_2}{2}\right)\left(\frac{b_1+b_2}{2}\right)$$

$m = (a_1b_1, a_1b_2, a_2b_{1,} a_2, b_2$,

$n = (a_1 - h)(b_1 - k).(a_1 - h)(b_1 + k).(a_2 + h)(b_1 - k).(a_2 + h)(b_2 + k)$,

$n' = (a_1 - h')(b_1 - k').(a_1 - h')(b_1 + k').(a_2 + h')(b_1 - k').(a_2 + h')(b_2 + k')$

iv) Division: $\tilde{A}^{I} \times \tilde{B}^{I} = (\frac{a_1 + a_2}{b_1 + b_2} - w, \frac{a_1 + a_2}{b_1 + b_2} + .|w - w'|.|w - w_1'|)$

Where $w = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(m), \max(m) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\}$,

$w' = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(n), \max(n) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\}$,

$w'_1 = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(n'), \max(n') - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\}$,

$m = \left(\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right)$, $n = \left(\left[\frac{a_1 - h}{b_1 - k}\right], \left[\frac{a_1 - h}{b_2 + k}\right], \left[\frac{a_2 + h}{b_1 - k}\right], \left[\frac{a_2 + h}{b_2 + k}\right]\right)$,

$n' = \left(\left[\frac{a_1 - h'}{b_1 - k'}\right], \left[\frac{a_1 - h'}{b_2 + k'}\right], \left[\frac{a_2 + h'}{b_1 - k'}\right], \left[\frac{a_2 + h'}{b_2 + k'}\right]\right)$,

Definition: 2.4. Let F(S) be the set of all Symmetric trapezoidal intuitionistic fuzzy numbers. For $\tilde{A}^{I} = \{(a_1, a_2, h, h); (a_1, a_2, h', h')\} \in \Re(s)$ we define a ranking function $F : F(s) \to \Re$ by $F(\tilde{A}^{I} = \frac{(a_1 + a_2}{2} + (h - h')$.

III. INTUITIONISTIC FUZZY LINEAR COMPLEMENTARITY PROBLEM (IFLCP)

3.1. Fuzzy Linear Complementarity Problem (FLCP)

Assume that all parameters in (1) - (3) are fuzzy and are described by fuzzy numbers. Then, the following fuzzy linear complementarity problem can be obtained by replacing crisp parameters with fuzzy numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \quad (1)$$

$$\tilde{W}_j \geq 0, Z_j \geq 0, j = 1, 2, 3, ...n \quad (2)$$

$$\tilde{W}_j \tilde{Z}_j = 0, j = 1, 2, 3, ...n \quad (3)$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy complementary variables.

3.2 Intuitionistic Fuzzy Quadratic Programming Problem (QPP) as a Intuitionistic fuzzy Linear Complimentarity Problem (LCP)

Consider the following Quadratic Programming Problem

Minimize $\tilde{f}^{I}(\tilde{x})^{I} = \tilde{c}^{tI}\tilde{x}^{I} + \frac{1}{2}\tilde{x}^{tI}\tilde{H}^{I}\tilde{x}^{I}$ Subject to $\tilde{A}^{I}\tilde{x}^{I} \leq \tilde{b}^{I}$,and $\tilde{x}^{I} \geq 0$

Where $\tilde{c}$ an n-vector of fuzzy numbers is, $\tilde{b}$ is an m-vector, Ã is an mxn fuzzy matrix and $\tilde{H}$ is an nxn fuzzy symmetric matrix. Let $\tilde{y}$ denotes the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian multiplier vectors of the constraints Ã $\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ respectively, then the Kuhn-Tucker conditions can be written as

$\tilde{A}^{I}\tilde{x}^{I}+\tilde{y}^{I}=\tilde{b}^{I}$ - $\tilde{H}^{I}\tilde{x}^{I}-\tilde{A}^{tI}\tilde{u}^{I}+\tilde{v}^{I}=\tilde{c}^{I}$ $\tilde{x}^{tI}\tilde{v}^{I}=\tilde{0}^{I},\tilde{u}^{tI}\tilde{y}^{I}=\tilde{0}^{I}$ And $\tilde{x}^{I}$, $\tilde{y}^{I}$ $\tilde{u}^{I},\tilde{v}^{I}\geq\tilde{0}^{I}$ Now Letting $\tilde{M}^{I}=\begin{bmatrix}\tilde{0}^{I} & -\tilde{A}^{I}\\ \tilde{A}^{tI} & \tilde{H}^{I}\end{bmatrix},\tilde{q}^{I}=\begin{bmatrix}\tilde{b}^{I}\\ \tilde{c}^{I}\end{bmatrix},\tilde{w}^{I}=\begin{bmatrix}\tilde{y}^{I}\\ \tilde{v}^{I}\end{bmatrix}$ and $\tilde{z}^{I}=\begin{bmatrix}\tilde{u}^{I}\\ \tilde{v}^{I}\end{bmatrix}$ the Kuhn-Tucker conditions can be expressed as the LCP. $\tilde{W}^{I}-\tilde{M}^{I}\tilde{Z}^{I}=\tilde{q}^{I},\tilde{W}^{tI}\tilde{Z}^{I}=\tilde{0}^{I}$, $(\tilde{W}^{I},\tilde{Z}^{I})\geq\tilde{0}^{I}$

IV. IMPLEMENTATION OF THE COMPLEMENTARY PIVOT METHOD USING THE INVERSE OF THE BASIS FOR SOLVING FUZZY LINEAR COMPLEMENTARITY PROBLEM

Step 1: Introduce the fuzzy artificial variable associated $\tilde{z}^{I}{}_{0}$ with the column vector I_t for the purpose of obtaining a feasible basis.

Step 2: Identify row t such that $\tilde{q}_t^{\,I}$ = minimum { $\tilde{q}_i^{\,I}$; $1\leq i\leq n$. Break ties for t in this equation arbitrarily. Since we assumed $\tilde{q}_t^{\,I}<0$. When a pivot is made with the column vector of $\tilde{z}^{I}{}_{0}$ as the pivot column and the t^th^ row as the pivot row, the right hand side constant vectors becomes a non negative vector. Therefore, here the initial basic vector is $(\tilde{W}_1^{\,I},.........\tilde{W}_{t-1}^{\,I},\tilde{z}_0^{\,I},\tilde{W}_{t+1}^{\,I},....\tilde{W}_n^{\,I})$

Step 3: After performing the pivot with row t as the pivot row and the column vector $\tilde{z}^{I}{}_{0}$ of as the pivot column, we get the initial tableau for this algorithm.

Step 4: Let P_0 be the pivot matrix of order n obtained by replacing the t^{th} column in I (the unit matrix of order n) by $-e_n$ (the column vector in R_n all of whose entries are -1).Let
M' = P_0 M, q' = $P_0\tilde{q}^{I}$. Then the initial tableau in this algorithm is

$\tilde{W}^{I}$	$\tilde{z}^{I}$	$\tilde{z}^{I}{}_{0}$	
P_O	-M′	$I_{.T}$	$\tilde{q}^{I}$

The Initial Basic Vector is $(\tilde{W}_1^{\,I},.........\tilde{W}_{t-1}^{\,I},\tilde{z}_0^{\,I},\tilde{W}_{t+1}^{\,I},....\tilde{W}_n^{\,I})$

Step 5: Let B be the basis from (4.1), corresponding to the present basic Vector. Let β = β_{ij} = B^{-1} Le t β = ($_{ij}$) = B^{-1} and $\tilde{q}$ = B^{-1} q^{1} . Then the inverse tableau is

BASIC VECTOR	INVERSE	
	B = B_{IJ} = B^{-}	$\tilde{q}^{I}$

Step 6: To find the entering variable. The updated column of $\tilde{Ys}^{I}$ is βP_0 $I_{.s}$ if $\tilde{Ys}^{I}=\tilde{W}_s^{\,I}$

Suppose this pivot column is $(\tilde{a}_{1s}^{\,I},..........\tilde{a}^{I}{}_{ns})^{T}\leq 0.$ We have ray termination and the method has been unable to solve this IFLCP. Otherwise go to next step.

Step 7: To find the leaving variable. The minimum ratio is $\theta=\min\left\{\dfrac{\tilde{q}^{I}{}_{i}}{\tilde{a}^{I}{}_{is}}\quad i;\tilde{a}^{I}{}_{is}\geq 0\right.$

If the i that attains this minimum is unique, it determines the pivot row uniquely. The present basic variable in the pivot row is the leaving variable. Suppose i do not uniquely, check whether $\tilde{z}^{I}{}_{0}$ is eligible to drop and if so choose it as the leaving variable. If $\tilde{z}^{I}{}_{0}$ is not eligible to drop, we can be choosen arbitrarily. Once the leaving variable is identified, performing the pivot leads to the next basis inverse, and the entering variable in the next step is the complement of the leaving variable, and the method is continued in the same way.

V.ILLUSTRATIVE EXAMPLE

Consider the LCP $(\tilde{q}, \tilde{M})$ with intuitionistic triangular fuzzy number is

$$\tilde{M}^I = \begin{bmatrix} \tilde{1}^I & \tilde{0}^I & \tilde{0}^I \\ \tilde{2}^I & \tilde{1}^I & \tilde{0}^I \\ \tilde{2}^I & \tilde{2}^I & \tilde{1}^I \end{bmatrix} \text{ and } \tilde{q}^I = \begin{bmatrix} \tilde{6}^I \\ \tilde{2}^I \\ \tilde{14}^I \end{bmatrix}$$

For Solving the above Intuitionistic fuzzy Linear Complimentarity problem $(\tilde{q}, \tilde{M})$ by Implementation of the Complementary Pivot Method Using the Inverse of the Basis

FIG:I

BASIC VECTOR	INVERSE			$\tilde{q}^I$	PIVOT COLUMN $\tilde{z}^I{}_3$	RATIO
$\tilde{W}_1{}^I$	{(1,1,1); (.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(0,0,0); (0,0,0)}	{(3,6,9); (2.778,6,9.35)}	{(1,1,1); (.5,1,1.5)}	{(3,6,9); (2.778,6,9.35)}
$\tilde{W}_2{}^I$	{(0,0,0); (0,0,0)}	{(1,1,1); (.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(1,2,3); (.5,2,3.5)}	{(1,1,1); (.5,1,1.5)}	{(1,2,3); (.5,2,3.5)}
$\tilde{z}^I{}_0$	{(0,0,0); (0,0,0)}	{(0,0,0); (0,0,0)}	{(1,1,1); (.5,1,1.5)}	{(13,14,15); (12.8,14,15.92)}	{(1,1,1); (.5,1,1.5)}	{(13,14,15); (12.8,14,15.92)}

FIG:II

BASIC VECTOR	INVERSE			$\tilde{q}^I$	PIVOT COLUMN $\tilde{z}^I{}_2$	RATIO
$\tilde{W}_1{}^I$	{(1,1,1); (.5,1,1.5)}	-{(1,1,1); (.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(3,4,5); (2.79,4,5.8)}	{(1,1,1); (.5,1,1.5)}	{(3,4,5); (2.79,4,5.8)}
$\tilde{z}^I{}_1$	{(0,0,0); (0,0,0)}	{(1,1,1); (.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(1,2,3); (.5,2,3.5)}	{(1,1,1); (.5,1,1.5)}	{(1,2,3); (.5,2,3.5)}
$\tilde{z}^I{}_0$	{(0,0,0); (0,0,0)}	-{(1,1,1); (.5,1,1.5)}	{(1,1,1); (.5,1,1.5)}	{(11,12,13); (10.9,12,13.81)	{(1,1,1); (.5,1,1.5)}	{(11,12,13); (10.9,12,13.81)

Proceeding in this way and that the final table becomes

FIG:III

BASIC VECTOR	INVERSE			$\tilde{q}^I$	PIVOT COLUMN	RATIO
$\tilde{z}_1{}^I$	−{(1,1,1); (.5,1,1.5)}	{(0,0,0); (0,0,0)}	{(1,1,1); (.5,1,1.5)}	{(7,8,9); (6.7,8,9.9)		
$\tilde{w}_2{}^I$	-{(1,2,3); (.5,2,3.5)}	{(1,1,1); (.5,1,1.5)}	{(1,1,1); (.5,1,1.5)}	{(3,4,5); (2.5,4,5.6)}		
$\tilde{w}^I{}_3$	-{(1,2,3); (.5,2,3.5)}	{(0,0,0); (0,0,0)}	{(1,1,1); (.5,1,1.5)}	(1,2,3); (.5,2,3.5)}		

The Solution of the Intuitionistic fuzzy linear complimentarity is

$\tilde{w}_1{}^I = \{(0,0,0); (0,0,0)\}$, $\tilde{w}_2{}^I = \{(3,4,5); (2.5,4,5.6)\}$, $\tilde{w}_3{}^I = (1,2,3); (.5,2,3.5)\}$

$\tilde{z}_1{}^I = \{(7,8,9); (6.7,8,9.9)$, $\tilde{z}_2{}^I = \{(0,0,0); (0,0,0)\}$, $\tilde{z}_3{}^I = \{(0,0,0); (0,0,0)\}$

VI.CONCLUSION

In this paper, a new approach for solving an Intuitionistic fuzzy linear complementarity problem is suggested. Here, the implementation of the complementary pivot method using the inverse of the basis is proposed to solve the given fuzzy linear complementarity problem with the intuitionistic triangular fuzzy number gives the optimal solution of the given objective function is comparatively better to other methods. In future it can be extended into a real life applications with neutrosophic numbers approach.

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